

GRAVITATIONAL DUST COLLAPSE WITH COSMOLOGICAL CONSTANT

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Abstract

We study the fate of gravitational collapse in presence of a cosmological constant. The junctions conditions between static and non-static space-times are deduced. Three apparent horizon are formed, but only two have physical significance, one of them being the black hole horizon and the other the cosmological horizon. The cosmological constant term slows down the collapse of matter, limiting also the size of the black hole.

1 Introduction

The possible existence of a cosmological constant is one of the most important challenges in high energy physics today [1]. Quantum field theory predicts the existence of a huge vacuum energy density, of order of $\rho_V \sim 10^8 GeV^4$ (in units where $\hbar = c = 1$). Introducing it in the Einstein's field equation, this would lead to a overclosed Universe. To avoid this problem, we can include a cosmological constant term in the Einstein's equation, that would cancel exactly this contibution coming from the quantum vacuum. Of course, we must fine tune the constant term introduced by hand, presenting esthetical and conceptual drawbacks.

However, a surprising recent result coming from the analysis of high red-shift supernovae, indicates that the Universe may be accelerating now [2, 3, 4]. This suggests that there is in fact a cosmological constant, that dominates the content of energy of the Universe today. The cosmological implications of the existence of a cosmological constant today (we must remember that the inflationary scenario makes use extensively of a cosmological constant term in the primordial Universe) are enormous, concerning not only the evolution of the Universe but also the structure formation and age problems.

In any way, it is reasonable to think that under extreme physical conditions quantum effects may play an important role even in gravitating systems and the cosmological constant problem comes, under this circumstances, to surface. The gravitational collapse is one example of these extreme physical conditions where black holes seem to be formed. If we study dynamically the formation of a black hole, through a spherically symmetric spacetime in presence of dust matter, the matter suffers a gravitational collapse process, leading first to the formation of an apparent horizon, and then of a singularity. The mass, and consequently the dimension, of the black hole has not, in principle, any restrictions.

The goal of this work is to extend the study of the gravitational collapse of dust matter in the presence of a cosmological constant. This study has been made using static configuration for the spherically symmetric spacetime[5, 6]. We extend this analysis through the introduction of dynamics, considering the collapse process itself. Some of the results already found, like the limitation in size of a possible black hole, are reobtained. However, we can analyze dynamically the appearence of two physical apparent horizons, the slow down of the collapse process, giving some simple models to under-

stand why the cosmological constant, due to its repulsive character, limits the existence of possible black holes.

This article is organized as follows. In the next section, we consider the junctions conditions between a static and a non-static spherically symmetric spacetimes. In section 3, we solve the field equations using the Tolman-Bondi metric, with a matter content composed of dust and a cosmological constant. In section 4, we specialize the solution obtained in the previous section. The apparent horizons in such spacetimes are analyzed in section 5, and the consequences of the presence of a cosmological constant are considered. We end with the conclusions where we analyze and try to interpret our results.

2 Junction conditions

The object of this section is to derive the conditions to be satisfied at the surface of a collapsing perfect fluid sphere with cosmological constant. We assume spherical symmetry about an origin 0. Inside a spherical surface Σ , center 0, there exists a collapsing perfect fluid that can be described by the line element

$$ds^2 = dt^2 - X^2 dr^2 - Y^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

where X and Y are functions of r and t . Since we shall assume that the source is dust, which always moves along geodesics, the system used in (1) is considered comoving. For the exterior spacetime to Σ , since the fluid is not radiating, we have the Schwarzschild-de Sitter spacetime,

$$ds^2 = F dT^2 - \frac{1}{F} dR^2 - R^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (2)$$

where F is a function of R given by

$$F(R) = 1 - \frac{2M}{R} - \frac{\Lambda}{3} R^2, \quad (3)$$

where M is a constant and Λ the cosmological constant. In accordance with the Darmois junction conditions [7, 8] we suppose that the first and second fundamental forms inherited by Σ from the interior (1) and exterior (2) spacetimes are the same. The conditions are necessary and sufficient for a smooth matching. The equations of Σ may be written

$$r - r_\Sigma = 0 \text{ in } V^-, \quad (4)$$

$$R - R_\Sigma(T) = 0 \text{ in } V^+, \quad (5)$$

where V^- refers to the spacetime interior of Σ and V^+ to the spacetime exterior, and r_Σ is a constant because Σ is a comoving surface forming the boundary of the dust. To apply the junction conditions we must arrange that Σ has the same parametrisation whether it is considered as embedded in V^+ or in V^- . We have for the metric (1) using (4,5) on Σ ,

$$ds_\Sigma^2 = dt^2 - [Y(r_\Sigma, t)]^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (6)$$

We shall take $\xi^0 = t$, $\xi^2 = \theta$ and $\xi^3 = \phi$ as the parameters on Σ , hence the first fundamental form of Σ can be written,

$$g_{ij} d\xi^i d\xi^j, \quad (7)$$

with latin indices ranging 0,2,3 and g_{ij} the metric on Σ . The metric (2) considering (6) becomes on Σ

$$ds_\Sigma^2 = \left[F(R_\Sigma) - \frac{1}{F(R_\Sigma)} \left(\frac{dR_\Sigma}{dT} \right)^2 \right] dT^2 - R_\Sigma^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (8)$$

where we assume

$$F(R_\Sigma) - \frac{1}{F(R_\Sigma)} \left(\frac{dR_\Sigma}{dT} \right)^2 > 0, \quad (9)$$

so that T is a timelike coordinate. Now considering the continuity of the first fundamental form (7) from the metrics on Σ (6) and (7) we get

$$R_\Sigma = Y(r_\Sigma, t), \quad (10)$$

$$\left[F(R_\Sigma) - \frac{1}{F(R_\Sigma)} \left(\frac{dR_\Sigma}{dT} \right)^2 \right]^{1/2} dT = dt. \quad (11)$$

The second fundamental form of Σ is

$$K_{ij} d\xi^i d\xi^j, \quad (12)$$

where K_{ij} is the extrinsic curvature given on the two sides by

$$K_{ij}^\pm = -n_{ij}^\pm \frac{\partial^2 x^\alpha}{\partial \xi^i \partial \xi^j} - n_\alpha^\pm \Gamma_{\beta\gamma}^\alpha \frac{\partial x^\beta}{\partial \xi^i} \frac{\partial x^\gamma}{\partial \xi^j}. \quad (13)$$

The Christoffel symbols $\Gamma_{\beta\gamma}^\alpha$ are to be calculated from the appropriate interior or exterior metrics, (1) or (2), n_α^\pm are the outward unit normals to Σ in V^- and V^+ which come from (4) and (5)

$$n_\alpha^- = [0, X(r_\Sigma, t), 0, 0], \quad (14)$$

$$n_\alpha^+ = (-\dot{R}_\Sigma, \dot{T}, 0, 0), \quad (15)$$

where the dot stands for differentiation with respect to t , and x^α refers to the equation of Σ (4) and (5). The nonzero K_{ij}^\pm are the following

$$K_{22}^+ = \csc^2 \theta K_{33}^+ = \left(\frac{YY'}{X} \right)_\Sigma, \quad (16)$$

$$K_{00}^+ = \left(\dot{R}\ddot{T} - \dot{T}\ddot{R} - \frac{F}{2} \frac{dF}{dR} \dot{T}^3 + \frac{3}{2F} \frac{dF}{dR} \dot{T}\dot{R}^2 \right)_\Sigma, \quad (17)$$

$$K_{22}^+ = \csc^2 \theta K_{33}^+ = (FR\dot{T})_\Sigma. \quad (18)$$

The continuity of the second fundamental form imposes

$$K_{00}^+ = 0, \quad K_{22}^- = K_{22}^+, \quad (19)$$

on the nonzero components of the extrinsic curvature. Now by considering (19) with the conditions (10,11) and with (3) we have

$$(X\dot{Y}' - \dot{X}Y')_\Sigma = 0, \quad (20)$$

$$M = \left(\frac{Y}{2} - \frac{\Lambda}{6} Y^3 + \frac{Y}{2} \dot{Y}^2 - \frac{Y}{2X^2} Y'^2 \right)_\Sigma. \quad (21)$$

Let us now summarise the junction conditions of Σ . The necessary and sufficient conditions for smooth matching of metric (1), being comoving, and metric (2) are (10,11) and (20,21).

3 Field equations

The spherical dust collapse with cosmological constant is described by Einstein's field equations,

$$R_{\alpha\beta} = 8\pi\rho \left(u_\alpha u_\beta - \frac{1}{2} g_{\alpha\beta} \right) - \Lambda g_{\alpha\beta}, \quad (22)$$

where ρ is the energy density. Considering the line element 1) and numbering the coordinates $x^0 = t$, $x^1 = r$, $x^2 = \theta$ and $x^3 = \phi$ we obtain for the components of (22), with the comoving four velocity $u_\alpha = \delta_\alpha^0$,

$$R_{00} = -\frac{\ddot{X}}{X} - 2\frac{\ddot{Y}}{Y} = 4\pi\rho - \Lambda, \quad (23)$$

$$R_{11} = \frac{\ddot{X}}{X} + 2\frac{\dot{X}\dot{Y}}{XY} - \frac{2}{X^2} \left(\frac{Y''}{Y} - \frac{X'Y'}{XY} \right) = 4\pi\rho + \Lambda, \quad (24)$$

$$R_{22} = \frac{\ddot{Y}}{Y} + \left(\frac{\dot{Y}}{Y} \right)^2 + \frac{\dot{X}\dot{Y}}{XY} - \frac{1}{X^2} \left[\frac{Y''}{Y} + \left(\frac{Y'}{Y} \right)^2 - \frac{X'Y'}{XY} - \left(\frac{X}{Y} \right)^2 \right] \\ = \csc^2 \theta R_{33} = 4\pi\rho + \Lambda, \quad (25)$$

$$R_{01} = -2\frac{\dot{Y}'}{Y} + 2\frac{\dot{X}}{X} \frac{Y'}{Y} = 0. \quad (26)$$

Integrating (26) we obtain

$$X = \frac{Y'}{W}, \quad (27)$$

where $W = W(r)$ is an arbitrary function of r . From (23, 24,25) with (27) we obtain

$$2\frac{\ddot{Y}}{Y} + \left(\frac{\dot{Y}}{Y} \right)^2 + \frac{1 - W^2}{Y^2} = \Lambda, \quad (28)$$

and after integration,

$$\dot{Y}^2 = W^2 - 1 + 2\frac{m}{Y} + \frac{\Lambda}{3}Y^2, \quad (29)$$

where $m = m(r)$ is an arbitrary function of r . Substituting (27,29) into either of (23,24,25) we obtain

$$m' = 4\pi\rho Y^2 Y'. \quad (30)$$

Integrating (30) we obtain

$$m(r) = 4\pi \int_0^r \rho Y^2 dY + m_0, \quad (31)$$

where m_0 is an arbitrary constant. Since we want a finite distribution of matter at the origin $r = 0$ we assume $m_0 = 0$. From the field equation (26)

we see that the junction condition (20) is identically satisfied. Substituting into the junction condition (21) equations (3,27,29) we get

$$M = m_{\Sigma}. \quad (32)$$

From (3) we see that if $\Lambda = 0$ the exterior spacetime becomes the Schwarzschild spacetime and M is interpreted as the total energy inside Σ because of its Newtonian asymptotic behaviour. To calculate the total energy $\mathcal{M}(r, t)$ up to a radius r at a time t inside Σ we use the definition of mass function [9, 10, 11] which is proportional to the component of the Riemann tensor R^{23}_{23} , and is given for the metric (1) as

$$\mathcal{M}(r, t) = \frac{1}{2}Y^3 R^{23}_{23} = \frac{1}{2}Y \left[1 - \left(\frac{Y'}{X} \right)^2 + \dot{Y}^2 \right]. \quad (33)$$

Using (27,29) into (33), we get

$$\mathcal{M}(r, t) = m(r) + \frac{\Lambda}{6}Y^3(r, t). \quad (34)$$

The quantity $m(r)$ can be interpreted as energy due to the energy density $\rho(r, t)$, given by (31), and since it is measured in a comoving frame m is only r dependent. If we give a perfect fluid interpretation to the cosmological constant Λ like

$$\Lambda = 8\pi\mu, \quad (35)$$

where μ is a constant energy density and with a pressure $p = -\mu$, then the second term in the right hand side of (34) becomes,

$$\frac{\Lambda}{6}Y^3 = \frac{4\pi}{3}\mu Y^3. \quad (36)$$

Since the related cosmological fluid is not comoving with the frame inside Σ , its associated energy is time dependent. Its contribution to the total energy is positive or negative accordingly if $\Lambda > 0$ or $\Lambda < 0$. Another expression for the total energy inside Σ , for slow collapse, has been proposed by Tolman [15] and Whittaker [16] (for a discussion about its physical meaning see [11]). That expression reduces, in our case, to (31). When shells of matter cross each other, there are shell crossing singularities [12] and to avoid them we need

that the proper radius increases with the coordinate r , hence we require from (27)

$$X(r, t) > 0. \quad (37)$$

Since we assume $\rho(r, t) > 0$ then (30) gives

$$m' \geq 0, \quad (38)$$

which means that the energy density increases with r .

4 Dust solution with $W(r) = 1$

From now on we consider only the case $\Lambda > 0$ and the assumption

$$W(r) = 1. \quad (39)$$

From (27), with (39), both the radii of the collapsing sphere, calculated from g_{11} and from g_{33} (which follows from the perimeter divided by 2π), are the same. The condition (39) in the absence of cosmological constant is the marginally bound condition, limiting the situations where the shell is bounded from those it is unbounded. In the presence of a cosmological constant, the situation is more complex, and $W(r) = 1$ leads to an unbounded shell. When $W(r) < 1$, there is unbounded shell for $\Lambda > \Lambda_c$ and bounded for $\Lambda < \Lambda_c$, Λ_c being the root of (29). Then with (39) we obtain from (27,29),

$$Y(r, t) = \left(\frac{6m}{\Lambda}\right)^{1/3} \sinh^{2/3} \alpha(r, t), \quad (40)$$

$$X(r, t) = \left(\frac{6m}{\Lambda}\right)^{1/3} \left[\frac{m'}{3m} \sinh \alpha(r, t) + \left(\frac{\Lambda}{3}\right)^{1/2} t'_0 \cosh \alpha(r, t) \right] \sinh^{-1/3} \alpha(r, t), \quad (41)$$

where

$$\alpha(r, t) = \frac{\sqrt{3\Lambda}}{2} [t_0(r) - t], \quad (42)$$

and $t_0(r)$ is an arbitrary function of r . For $t = t_0(r)$ we have $Y(r, t) = 0$ which is the time when the matter shell $r = \text{constant}$ hits the physical singularity.

Taking the limit $\Lambda \rightarrow 0$ of (40,41) we reobtain the Tolman-Bondi solution [13, 14]

$$\lim_{\Lambda \rightarrow 0} Y(r, t) = \left[\frac{9m}{2} (t_0 - t)^2 \right]^{1/3}, \quad (43)$$

$$\lim_{\Lambda \rightarrow 0} X(r, t) = \frac{m'(t_0 - t) + 2mt'_0}{[6m^2(t_0 - t)]^{1/3}}. \quad (44)$$

5 Apparent horizons

The apparent horizon is formed when the boundary of trapped two spheres are formed. We search for two spheres whose outward normals are null, which give for (1),

$$g^{\mu\nu} Y_{,\mu} Y_{,\nu} = -\dot{Y}^2 + \left(\frac{Y'}{X} \right)^2 = 0. \quad (45)$$

Considering (27,29) we have from (45),

$$\Lambda Y^3 - 3Y + 6m = 0 \quad (46)$$

where the solutions for Y give the apparent horizons. For $\Lambda = 0$ we have the Schwarzschild horizon $Y = 2m$, and for $m = 0$ we have the de Sitter horizon $Y = \sqrt{3/\Lambda}$. For $3m < 1/\sqrt{\Lambda}$ there are two horizons,

$$Y_1 = \frac{2}{\sqrt{\Lambda}} \cos \frac{\varphi}{3}, \quad (47)$$

$$Y_2 = -\frac{1}{\sqrt{\Lambda}} \left(\cos \frac{\varphi}{3} - \sqrt{3} \sin \frac{\varphi}{3} \right), \quad (48)$$

where φ is given by

$$\cos \varphi = -3m\sqrt{\Lambda}. \quad (49)$$

If $m = 0$ we have $Y_2 = 0$ and $Y_1 = \sqrt{3/\Lambda}$, then we call Y_1 the cosmological horizon generalized when $m \neq 0$, and Y_2 the black hole horizon generalized when $\Lambda \neq 0$ [6]. For $3m = 1/\sqrt{\Lambda}$ (47) and (48) coincide and there is only one horizon,

$$Y = \frac{1}{\sqrt{\Lambda}} \quad (50)$$

In general, the range for Y_1 and Y_2 is given by

$$0 \leq Y_2 \leq \frac{1}{\sqrt{\Lambda}} \leq Y_1 \leq \sqrt{\frac{3}{\Lambda}}. \quad (51)$$

The biggest amount of mass m for the formation of an apparent black hole horizon is given by (50), attaining at that stage its largest proper area $4\pi/Y^2(r, t) = 4\pi/\Lambda$. The cosmological horizon has an area between $4\pi/\Lambda$ up to $12\pi/\Lambda$. For $3m > 1/\sqrt{\Lambda}$ there are no horizons. The time for the formation of the apparent horizon, for $W(r) = 1$, can be obtained from (40) with (46) giving

$$t_n = t_0 - \frac{2}{\sqrt{3\Lambda}} \arg \sinh \left(\frac{Y_n}{2m} - 1 \right)^{1/2}, \quad (52)$$

where Y_n stands for either of the values (47,48) or (50). In the limit when $\Lambda = 0$ it is reobtained the Tolman-Bondi result [12],

$$t_{AH} = t_0 - \frac{4}{3}m. \quad (53)$$

From (51) we have that both apparent horizons, black hole and cosmological, precede the singularity $t = t_0$ by an amount of comoving time $(2/\sqrt{3\Lambda}) \arg \sinh \sqrt{(Y_n/2m) - 1}$. From (51) we can write

$$\frac{Y_n}{2m} = \cosh^2 \alpha_n. \quad (54)$$

Since, from (51), always $Y_1 \geq Y_2$, we have also from (53) $\alpha_1 \geq \alpha_2$ or $t_1 \leq t_2$, which means that the cosmological horizon always precedes the black hole horizon. To see how t_n behaves as a function of m , we first calculate, using (47,48,49),

$$\frac{d(Y_1/2m)}{dm} = \frac{1}{m} \left(-\frac{\sin \varphi/3}{\sin \varphi} + \frac{3 \cos \varphi/3}{\cos \varphi} \right) < 0, \quad (55)$$

$$\frac{d(Y_2/2m)}{dm} = \frac{1}{m} \left[-\frac{\sin(\varphi + 4\pi)/3}{\sin \varphi} + \frac{3 \cos(\varphi + 4\pi)/3}{\cos \varphi} \right] > 0. \quad (56)$$

Defining

$$\tau_n = t_0 - t_n, \quad (57)$$

we have from (54),

$$\frac{d\tau_n}{d(Y_n/2m)} = \frac{1}{\sqrt{\Lambda} \sinh \alpha_n \cosh \alpha_n}. \quad (58)$$

Now considering (55) and (47,58) we have

$$\begin{aligned} \frac{d\tau_1}{dm} &= \frac{d\tau_1}{d(Y_1/2m)} \frac{d(Y_1/2m)}{dm} \\ &= \frac{1}{m\sqrt{3\Lambda} \sinh \alpha_1 \cosh \alpha_1} \left(-\frac{\sin \varphi/3}{\sin \varphi} + \frac{3 \cos \varphi/3}{\cos \varphi} \right) < 0. \end{aligned} \quad (59)$$

From (59) we have that τ_1 decreases when m increases, which means that the time interval between the formation of the singularity and the cosmological horizon diminishes while m increases. While considering (48) and (56) we have

$$\begin{aligned} \frac{d\tau_2}{dm} &= \frac{1}{m\sqrt{3\Lambda} \sinh \alpha_2 \cosh \alpha_2} \\ &\times \left[-\frac{\sin(\varphi + 4\pi)/3}{\sin \varphi} + \frac{3 \cos(\varphi + 4\pi)/3}{\cos \varphi} \right] > 0. \end{aligned} \quad (60)$$

Hence from (60) we have that τ_2 decreases when m increases, meaning that the time interval between the formation of the singularity and the black hole horizon increases while m increases.

The nature of the surface of the two physical horizons, the cosmological and the black hole horizons, can be obtained calculating the induced metric on them. This can be done using

$$dt_n = \left(t'_0 - \frac{2}{\sqrt{3\Lambda}} \alpha'_n \right) dr \quad (61)$$

and introducing the roots for Y_n . For the cosmological horizon, we obtain that it is always past timelike. On the other hand, the nature of the black hole horizon depends crucially on the value of t'_0 . Considering $m' > 0$, we obtain,

$$0 \leq t'_0 \leq \frac{m'}{6m} \frac{Y_2(1 + \Lambda Y_2^2)}{1 - \Lambda Y_2^2} \quad (\text{past timelike}), \quad (62)$$

$$0 < t'_0 = \frac{m' Y_2(1 + \Lambda Y_2^2)}{6m(1 - \Lambda Y_2^2)} \quad (\text{past null}), \quad (63)$$

$$0 < \frac{m' Y_2(1 + \Lambda Y_2^2)}{6m(1 - \Lambda Y_2^2)} < t'_0 \quad (\text{spacelike}). \quad (64)$$

In the limit $\Lambda \rightarrow 0$, we find the results of [12].

6 Conclusions

In the collapse of a dust matter in a spherically symmetric spacetime, a black hole is inevitably formed. The singularity is preceded by the formation of an apparent horizon by a lapse of time $\Delta t = (4/3)m$, where m is the black hole mass. The dimension, consequently the mass of the black hole, is not limited in principle. In this analysis, it is employed the Tolman-Bondi metric which is asymptotically flat. In this case, the mass of the configuration is the same either if we use the Cahill-McVittie or the Tolman-Whittaker criteria.

The introduction of a cosmological constant changes this scenario in many ways. There are now three apparent horizons instead of one. One of these apparent horizon is not physical, while the other two represent the black hole horizon and the cosmological horizon. The spacetime is no more asymptotically flat. The cosmological constant affects the lapse of time between the formation of the apparent horizon and the singularity.

The two main consequences of introducing the cosmological constant, however, concerns the mass definition and the dimension of the black hole. Using the different definitions of mass in a gravitating system in a curved background can lead to a mass that includes or not the energy associated with the cosmological constant. Moreover, a black hole with a mass that exceeds $m = 1/\sqrt{\Lambda}$ can not be formed. This result has already been obtained in a static spherically symmetric configuration; but it remains when we introduce dynamics in the system.

In general, the introduction of a cosmological constant slows down the collapse process. The cosmological constant leads, when we think in terms of a Newtonian potential, to a repulsive term. Taking the g_{00} in the case of a static symmetric spacetime in presence of a cosmological constant, and considering that $g_{00} = 1 - 2\phi(R)$, where $\phi = (m/R) + (1/6)\Lambda R^2$, so that the

Newtonian force is given by

$$F(R) = -\frac{m}{R^2} + \frac{\Lambda}{3}R \quad . \quad (65)$$

For $R = 1/\sqrt{\Lambda}$ and $m = (1/3)\sqrt{\Lambda}$ (65) leads to $F = 0$. Mass and radius, larger than those ones, the force becomes repulsive, while for smaller values of mass and radius, the force becomes attractive, leading to the collapse and formation of a singularity. In this sense, the limitation of the size of the black hole can be understood in terms of this competition between repulsive and attractive forces, where the first comes from the cosmological constant term.

The slow down of the collapse process can be verified from the expression

$$\ddot{Y} = -\frac{m}{Y^2} + \frac{\Lambda}{3}Y, \quad (66)$$

obtained from (28,29). This expression reproduces the Newtonian model formulated before: the presence of a cosmological constant diminishes the acceleration in the collapse process, and for $Y \geq (3m/\Lambda)^{1/3}$, there is no collapse at all, since the acceleration becomes positive.

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